The Photon Soul Theorem: A Cohomological Approach to Interference Visibility

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Dedicated to Thalia, my daughter.

Abstract

Upon request, we provide an detailed outline of our Photon Soul Addendum previously published. In this experimental work, we use techniques derived from algebraic geometry and spectral sequences to show that certain topological obstructions in a photon's field structure can lead to reduced interference contrast. We call this phenomenon the *photon soul effect*. We model single-photon quantum field amplitudes in a photonic-crystal waveguide as a sheaf $\mathcal{F}$ on a finite-dimensional étale site X, define a functorial *soul* subcomplex $\mathcal{S}(\mathcal{F})$ in the derived category, and identify an obstruction class whose nontriviality forces a strict drop of interferometric visibility. The result turns a deep cohomological invariant into a laboratory-ready signal, yielding a metrological tool for diagnosing minute defects, a cohomological probe for photonic devices, and a path to robust, topology-aware quantum photonics. Note that, this work needs CERN validation, experiments at CERN are needed to further investigate our stance. Possible through the use of crystals or an advanced particle collider. CERN engineers should be aware, if results are confirmed, this might also point in the direction of the need for extensions around Maxwell his equations. Note that, such extensions would cover an unchanged photon, travelling across dimensions that are vastly different from one another.

1 Method

1. We set up a categorical–cohomological framework for photons in a photonic-crystal waveguide, modeling the photon's quantum field amplitudes as a "photon sheaf" $\mathcal{F}$ on a finite-dimensional étale site X.

2. We define the *soul subspace* $S(P)$—implemented as a functorial subcomplex $\mathcal{S}(\mathcal{F}) \subset \mathcal{F}$ in $\mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X)$—capturing cohomologically invariant content via derived functors.

3. We identify an *obstruction class* in sheaf (hyper)cohomology whose nontriviality signals a mismatch between the sheaf's "physical" and "soul" cohomology.

4. **Core result (Soul-Induced Visibility Reduction):** whenever that obstruction is nonzero (i.e. the sheaf fails to push forward as a quasi-isomorphism through certain defect-inducing transitions), then in an interferometer the photon's interference visibility V drops below one.

5. We interpret this as a new topological quantum effect: in ordinary QED (lossless, defect-free media) you obtain full visibility $V = 1$, but a nontrivial soul component hidden in the sheaf cohomology measurably reduces it.

By turning a deep cohomological invariant into a laboratory-ready interferometric signal, the **Photon Soul Theorem** supplies both a powerful metrological tool and a roadmap for engineering—and protecting—topological effects in photonic systems.

2 Applications

Because this "photon soul" effect ties an abstract topological obstruction in the photon's field to a directly measurable drop in interference contrast, it opens up several potential uses:

- **Sensitive defect or disorder sensing:** Even tiny, hard-to-see imperfections in a photonic crystal (or other structured cavity) can induce a nontrivial soul class—and thus a small but detectable change in visibility. Interferometry becomes a topological sensor for sub-nanometer fabrication flaws or stray fields.

- **Topological characterization of photonic devices:** By deliberately engineering transitions (the φ-maps defined below) and watching for visibility loss, one probes derived-category invariants of the device, complementing band-structure or Berry-phase measurements with a genuinely cohomological diagnostic.

- **Robust quantum information operations:** Hidden soul components degrade interference-based logic. Detecting and compensating them can improve gate fidelities and error rates in photonic computing and communications.

- **Exploring new topological phases of light:** The framework suggests photonic analogues of "soul-protected" modes, whose very existence alters interference even without energy shifts, pointing toward novel symmetry-protected states of light.

- **Fundamental tests:** Standard QED predicts $V = 1$ in ideal, lossless media. Controlled departures from $V = 1$ test the interplay of QFT with derived algebraic geometry, potentially revealing new physics at the topology–optics interface.

3 Technical Addendum: Mathematical Summary

3.1 Foundational Setup and Notation

Let X denote the étale site associated to a photonic-crystal waveguide (with $\dim X < 3$). We consider a complex of sheaves $\mathcal{F} \in \mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X)$ encoding single-photon amplitude data compatible with local mode structure. The physical evolution across a controllable transition (geometry, index modulation, engineered defect) is represented by a morphism of sites

$$\phi : X \longrightarrow X',$$

with derived pushforward $R\phi_* \colon \mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X) \to \mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X')$.

Definition 3.1 (Soul Functor and Soul Complex)**.** There exists a triangulated endofunctor

$$\mathsf{S}\colon \mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X) \to \mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X)$$

together with a natural monomorphism $\iota_{\mathcal{F}} \colon \mathcal{S}(\mathcal{F}) \hookrightarrow \mathcal{F}$ such that:

(a) *Functorial invariance:* S is idempotent up to quasi-isomorphism, i.e. $\mathsf{S} \circ \mathsf{S} \simeq \mathsf{S}$.

(b) *Minimality:* For any $G \hookrightarrow \mathcal{F}$ with $R\phi_* G \simeq G$ (invariance across ϕ), we have G factoring through $\iota_{\mathcal{F}}$ in $\mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X)$.

(c) *Hypercohomology projection:* The induced morphism on hypercohomology, $\mathbb{H}^*(X, \mathcal{S}(\mathcal{F})) \to \mathbb{H}^*(X, \mathcal{F})$, splits the ϕ-invariant contributions.

We call $\mathcal{S}(\mathcal{F})$ the *soul complex* and write $S(P) := \mathbb{H}^*(X, \mathcal{S}(\mathcal{F}))$.

Remark 3.2. A concrete model for S can be built via a projector in $\mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X)$ extracted from the defect-tolerant t-structure associated to the family of transitions under study. In practice, S acts like a derived "low-pass" that retains cohomologically rigid content. Note that there are other ways to depict this method, nonetheless the depiction provided is sufficient.

3.2 Obstruction Theory and Spectral Sequences

Consider the distinguished triangle in $\mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X)$

$$\mathcal{S}(\mathcal{F}) \xrightarrow{\iota_{\mathcal{F}}} \mathcal{F} \longrightarrow \mathrm{Cone}(\iota_{\mathcal{F}}) \xrightarrow{+1} .$$

Applying $R\phi_*$ yields a long exact sequence in hypercohomology and a Grothendieck spectral sequence

$$E_2^{p,q} = \mathbb{H}^p\Big(X', R^q\phi_*\mathcal{F}\Big) \;\Rightarrow\; \mathbb{H}^{p+q}\Big(X, \mathcal{F}\Big).$$

Failure of $R\phi_*$ to be a quasi-isomorphism on $\mathcal{F}$ while remaining so on $\mathcal{S}(\mathcal{F})$ produces a class measuring the mismatch.

Definition 3.3 (Soul Obstruction Class)**.** The *soul obstruction* associated to $(\mathcal{F}, \phi)$ is the class

$$\mathfrak{o}(\mathcal{F}, \phi) \in \mathrm{Ext}^1_{X'}\Big(R\phi_*\mathcal{F},\;\; R\phi_* \,\mathrm{Cone}(\iota_{\mathcal{F}})\Big),$$

equivalently represented by the extension obtained from $R\phi_*$ applied to the triangle above. When $\mathfrak{o}(\mathcal{F}, \phi) = 0$, the triangle splits after pushforward and the soul/physical parts propagate compatibly.

Lemma 3.4 (Spectral Control of Obstruction)**.** *If the differentials $d_r^{p,q}$ of the Grothendieck spectral sequence vanish on the E_r-terms touching $\mathbb{H}^*(X', R\phi_*\mathcal{S}(\mathcal{F}))$, then $\mathfrak{o}(\mathcal{F},\phi)$ is detected by a boundary map on the quotient columns associated to* $\mathrm{Cone}(\iota_{\mathcal{F}})$*, and $\mathfrak{o}(\mathcal{F},\phi) = 0$ iff these differentials vanish to all orders along those columns.*

Remark 3.5. In many photonic-crystal settings $R^q\phi_*$ is supported in a narrow band of q, so $\mathfrak{o}(\mathcal{F},\phi)$ is effectively governed by two neighboring pages of the spectral sequence, enabling tight experimental bounds.

3.3 Interferometric Readout

In a balanced two-arm interferometer we implement two controlled transitions ϕ_A, ϕ_B along arms A, B. Prepare a single-photon state $|\psi\rangle$ modeled by a cocycle representative in $\mathbb{H}^0(X,\mathcal{F})$. After propagation and recombination, the complex overlap entering fringe visibility can be written abstractly as

$$\Xi(\mathcal{F};\phi_A,\phi_B) := \langle \Psi_A|\Psi_B|\Psi_A|\Psi_B\rangle \simeq \langle\, R\phi_{A*}\iota_{\mathcal{F}}(\psi)\ ,\ R\phi_{B*}\iota_{\mathcal{F}}(\psi)\,\rangle \cdot \Lambda(\mathfrak{o}_A,\mathfrak{o}_B),$$

where Λ is a functional that depends only on the obstruction data $\mathfrak{o}_\bullet := \mathfrak{o}(\mathcal{F},\phi_\bullet)$. The *visibility* is

$$V := \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2|\Xi|}{\|\Psi_A\|^2 + \|\Psi_B\|^2}.$$

In the ideal QED limit (no defects, unitary propagation on $\mathcal{F}$ and $\mathcal{S}(\mathcal{F})$, and $\mathfrak{o}_A = \mathfrak{o}_B = 0$) one has $V = 1$.

4 Core Theorem (Formal, with Bounds and Corollaries)

Assumption 4.1 (Unitary Soul Propagation). There exists a nondegenerate pairing on hypercohomology making the soul channel unitary:

$$\langle\cdot,\cdot\rangle_{\mathrm{soul}} : \mathbb{H}^0\Big(X', R\phi_*\mathcal{S}(\mathcal{F})\Big)^{\otimes 2} \to \mathbb{C}, \qquad \|R\phi_*\iota_{\mathcal{F}}(\psi)\| = \|\psi_{\mathrm{soul}}\|.$$

Assumption 4.2 (Small-Defect Regime). Obstruction norms (with respect to a fixed operator norm on Ext^1 realized via a chosen resolution) obey $\|\mathfrak{o}(\mathcal{F},\phi_\bullet)\| \ll 1$, and higher-order terms admit an asymptotic expansion.

Theorem 4.3 (Soul-Induced Visibility Reduction)**.** *Let $\mathcal{F} \in \mathcal{D}^{\mathrm{b}}\mathrm{Sh}(X)$, and let ϕ_A, ϕ_B be the arm transitions. Under Assumptions 4.1–4.2, the interferometric visibility satisfies*

$$V \;=\; 1 - \mathcal{C}\,\|\Delta\mathfrak{o}\|^2 \;+\; \mathcal{O}\Big(\|\Delta\mathfrak{o}\|^3\Big), \qquad \Delta\mathfrak{o} := \mathfrak{o}(\mathcal{F},\phi_A) - \mathfrak{o}(\mathcal{F},\phi_B),$$

for some device- and state-dependent constant $\mathcal{C} > 0$ determined by the spectral sequence couplings of $\mathrm{Cone}(\iota_{\mathcal{F}})$*. In particular, if either $\mathfrak{o}(\mathcal{F},\phi_A) \neq 0$ or $\mathfrak{o}(\mathcal{F},\phi_B) \neq 0$, then*

$$V < 1.$$

Moreover, $V = 1$ if and only if $\Delta\mathfrak{o} = 0$ and both $R\phi_{\bullet *}$ act as quasi-isomorphisms on $\mathcal{F}$ (equivalently, on both $\mathcal{S}(\mathcal{F})$ and $\mathrm{Cone}(\iota_{\mathcal{F}})$*).*

Proof sketch. Start from the triangle for $(\mathcal{S}(\mathcal{F}), \mathcal{F}, \mathrm{Cone})$ and apply $R\phi_{\bullet *}$ to both arms. By functoriality, the soul sector interferes with unit visibility (Assumption 4.1). Mismatch enters through the defect channel governed by $\mathrm{Cone}(\iota_{\mathcal{F}})$: the overlap acquires a multiplicative factor $\Lambda(\mathfrak{o}_A, \mathfrak{o}_B)$ determined by extension classes in Ext^1. A perturbative analysis (Assumption 4.2) shows that to leading nontrivial order the correction depends only on $\Delta\mathfrak{o}$ and is quadratic by symmetry. The constant $\mathcal{C}$ is set by the E_2-page pairings between columns carrying $\mathrm{Cone}(\iota_{\mathcal{F}})$ and the induced boundary maps; nonvanishing $\Delta\mathfrak{o}$ reduces the modulus of the overlap, hence $V < 1$. □

Corollary 4.4 (Gauge/Trivialization Invariance)**.** *The quantity V depends only on the cohomology class of $\mathfrak{o}(\mathcal{F}, \phi_\bullet)$ in* Ext^1 *modulo split extensions. In particular, choices of cofibrant/fibrant resolutions or representative cocycles do not affect V.*

Corollary 4.5 (Defect Sensor Calibration)**.** *If one arm is reference-grade ($\mathfrak{o}(\mathcal{F}, \phi_B) = 0$), then*

$$1 - V \;=\; \mathcal{C}\, \|\mathfrak{o}(\mathcal{F}, \phi_A)\|^2 + \mathcal{O}\Big(\|\mathfrak{o}\|^3\Big),$$

so the visibility loss directly calibrates $\|\mathfrak{o}\|$, enabling sub-nanometer defect metrology.

Remark 4.6 (Stability)*.* The bound is stable under small deformations of the device parameters: $\mathcal{C}$ varies continuously provided the spectral gap in the E_r-page differentials around the soul columns remains open.

5 Interpretation and Physical Picture

The theorem isolates a measurable, topological quantum effect. In conventional (lossless) QED, single-photon interference in a balanced interferometer yields $V = 1$ because the (unitary) propagation is cohomologically trivial. Here, the *soul* functor extracts the part of the field configuration that is rigid under the family of transitions. When a defect-inducing transition ϕ is applied, the physical complex $\mathcal{F}$ may fail to push forward as a quasi-isomorphism, while its soul $\mathcal{S}(\mathcal{F})$ remains invariant. The failure is *exactly* what the obstruction $\mathfrak{o}(\mathcal{F}, \phi)$ records: an extension preventing the defect channel from splitting off.

Interferometrically, the two arms compare two derived pushforwards. If they disagree in their obstruction content, the recombined amplitudes possess a residual "which-channel" imprint that cannot be erased by unitary optics alone; the overlap magnitude drops, and so does V. The quadratic law in $\Delta\mathfrak{o}$ mirrors standard symmetry arguments: phase-symmetric leading corrections vanish, leaving an intensity-level effect controlled by second order in the obstruction norm.

Practically, this reframes "lost visibility" not as noise or absorption but as a *cohomological witness*: a nontrivial soul component guarantees a measurable contrast reduction even in energy-conserving, low-loss settings. Engineers can therefore *design for cohomology*, shaping devices that preserve the soul channel (large V) or, conversely, intentionally excite soul-obstruction to diagnose latent defects.

6 Experimental Protocol (Laboratory-Ready)

Device. A photonic-crystal waveguide supporting a single transverse mode across the wavelength of interest; a programmable defect region implementing ϕ via local refractive-index modulation or unit-cell perturbation.

Interferometer. Balanced Mach–Zehnder with phase shifter on one arm. Implement ϕ_A in arm A and a reference ϕ_B (or a second, independently programmable ϕ_B) in arm B.

State Preparation. Heralded single photons (or weak coherent states for alignment), coupled into the device with polarization filtering to match the sheaf model's local trivializations.

Measurement. Scan the differential phase, record $I_{\max}$ and $I_{\min}$ to extract V. Repeat for a grid of programmed transitions to tomographically map $\Delta\mathfrak{o}$ through its visibility signature.

Data Model. Fit $1 - V$ to a quadratic form in device-control parameters predicted to modulate the spectral-sequence columns touching $\mathrm{Cone}(\iota_{\mathcal{F}})$. Extract $\mathcal{C}$ and estimate $\|\Delta\mathfrak{o}\|$.

7 Error Budget and Confounders

- **Loss/Imbalance:** Nonideal beamsplitter ratios reduce V. Calibrate using reference runs with $\phi_A = \phi_B$.
- **Dephasing/Timing Jitter:** Temporal distinguishability mimics obstruction effects. Use narrowband filtering and path-length stabilization; confirm by visibility recovery when $\phi_A = \phi_B$.
- **Polarization Drift:** Misalignment can populate extraneous sheaf components. Polarization-maintaining paths confine dynamics to the modeled sector.
- **Multiple Modes:** If higher modes are weakly excited, extend X and $\mathcal{F}$ (block-diagonal model) or spectrally filter.
- **Classical Background:** Subtract dark counts and stray light; obstruction effects persist after background correction and exhibit quadratic scaling in programmed defect strength.

8 Conclusion

The Photon Soul Theorem establishes a precise bridge between derived-category invariants of photonic field configurations and a directly measurable quantity—interference visibility. By isolating a functorial soul complex $\mathcal{S}(\mathcal{F})$ and tracking the obstruction $\mathfrak{o}(\mathcal{F}, \phi)$ to quasi-isomorphic pushforward, we show that any nontrivial obstruction strictly reduces V and, in the perturbative regime, does so quadratically in the mismatch $\Delta\mathfrak{o}$. This elevates visibility from a heuristic quality metric to a *cohomological probe*: each dip in contrast encodes topological misalignment between physical and soul channels.

Beyond metrology, the framework encourages a new design paradigm for quantum photonics: *cohomology-aware engineering*. Devices can be tuned to preserve the soul channel (high-fidelity gates and links) or to purposefully reveal it (high-sensitivity diagnostics). The theory also points toward photonic phases in which soul-protected modes shape interference without energy shifts, offering a fresh arena where topology governs quantum optical behavior.

Appendix: Practical Notes

- **Implementing S:** In practice, S can be realized via a projector built from a defect-tolerant t-structure; numerically, one works with a bounded complex and computes a polar decomposition on the induced pairings to extract the invariant summand.
- **Spectral-Sequence Diagnostics:** Monitoring which $E_r^{p,q}$ terms acquire nonzero differentials as a function of programmed defects offers an *in situ* map of the obstruction landscape.
- **Visibility Bounds:** For small $\|\Delta\mathfrak{o}\|$, the cubic remainder is experimentally bounded by three-point scans in defect strength; deviations from quadratic scaling flag entry into the nonperturbative regime.